A Saskatchewan Mathematics Resource

The Saskatchewan Education Sector Strategic Plan
2020 - 2021

Table of Contents

*This resource is organized around the **ESSENTIALS OF MATH INSTRUCTION**. Each section has specific subsections that relate to the important components contained therein.*

This Table of Contents is interactive (Click on the section titles to go to each specific section.)

Acknowledgements

Acknowledgements:

The SaskMATH Reference Committee, Vetting Sub Committee and Working Group have been essential to the creation and philosophy of this document and resource. Under the direction of the Primary and Secondary Outcome Owners, they were able to collaboratively create this resource under very tight timelines and through the period of the COVID - 19 Pandemic that was felt across the province, the country, and the world. Thank you to the Boards of Education and the Directors of Education for supporting the participation of these individuals and for the tireless work and deep thinking that came to fruition in this resource.

Below is the list of the individuals whose contribution to this work will impact the high quality instruction and assessment, and ultimately, the success of students in this great province:

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Special Acknowledgements:

It is with great sadness and gratitude that we as the SaskMATH team extend our heartfelt thank you to the following individuals who have had significant impact on the work but who have passed away during the development phase of this resource:

Ed Varjassy: Passed away October 7, 2020.

Elder Albert Scott: Passed away January 13, 2021.

Message from the Outcome Owner(s):

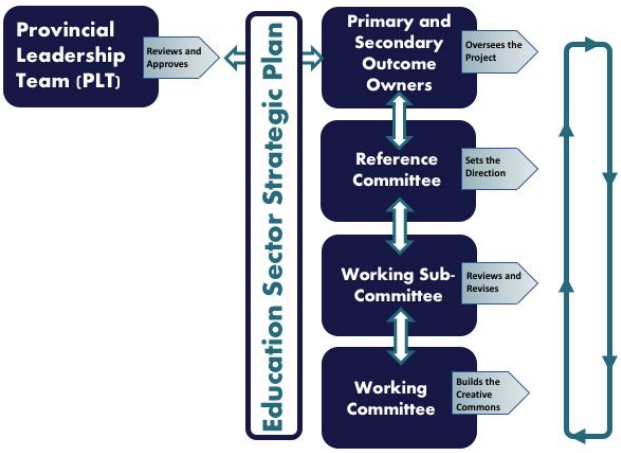
Do the best you can until you know better. Then when you know better, you do better[1]**.**

~ Maya Angelou

Sask MATH

Introduction:

This initiative originates with the Provincial Leadership Team and is part of the Math Outcome for Saskatchewan. It is intended to support teachers and leaders of mathematics in Saskatchewan by providing a framework of understanding around high quality instruction and assessment practices in Math. This diagram outlines the structure and processes for this outcome.



SaskMATH opens the opportunity to all students to know and appreciate Indigenous Ways of Knowing. I like the phrase used "coming to know" - everybody benefits from "coming to know Mathematics" and "coming to know Indigenous Ways of Knowing".

(Myrna Tuner, Cultural Coordinator for Muskoday First Nation Community School)

This resource follows the **Essentials of Math Instruction** as the foundation with each component contributing to the cohesive whole. It represents the implementation of the *Inspiring Success: First Nations and Métis PreK-12 Education Policy Framework*[1] document as it applies to mathematics with Indigenous Connections highlighted throughout each section in addition to one area dedicated to Indigenous Ways of Knowing in math. It is envisioned that through this important component, all students will benefit.

[1] Maya Angelou Quotes. (n.d.). allauthor.com. Retrieved December 22, 2020, from allauthor.com Web site: https://allauthor.com/quote/88789/

Visioning

SaskMATH Shared Beliefs

We believe that:

- high quality, data-informed, culturally appropriate, and student responsive math instruction is key to the life-long success of all children in Saskatchewan;
- providing all students with the foundations of Number Sense (compare, compose and decompose, represent, visualize, and estimate) lays the foundation for a deep understanding of relationships between and within numbers. This is the critical foundation for all outcomes, in all strands, and across all grade levels;
- all children can do math with respect to their personal experiences and needs; and
- engaging, effective, research-based, and culturally appropriate resources aid instructional decisions.

SaskMATH Mission

This SaskMATH web and print resource has been developed to support high quality math instruction and assessment that includes Indigenous Ways of Knowing. This resource is intended to support students, classroom teachers, administrators, families and caregivers, community partners, school division and First Nation Authority leaders, directors, school boards, and provincial policy-makers.

SaskMATH Vision

1. **WE will have** consistency and uniformity in the quality of instruction that will close achievement gaps province-wide where students have the numeracy and critical thinking skills needed to be successful.

2. **WE will have** teachers who have a deep level of understanding of content and pedagogical knowledge. That understanding will support a holistic understanding of the learner to provide responsive instructional decisions that meet the needs of each learner.

3. **WE will have** engaging, research-based, culturally and student responsive spaces for mathematics instruction.

4. **WE will have** students who will benefit from high quality instruction and formative assessment, as evidenced by data.

5. **WE will have** students at all levels working at their appropriate proficiency level as they work towards their mathematical potential.

Students are expected to ... develop a capacity for abstract thinking, which includes the critical thinking skills necessary for understanding global issues in society. [2]

6. **WE will have** students who enjoy, do and understand math, which will lead to confidence, ability, and a growth mindset as they use their knowledge and skill set beyond the classroom.

7. **WE will have** engaged students that have a sense of self-efficacy and persistence. Students will become mathematically literate citizens. Those students whose potential and interests direct them into further education and/or careers in science, technology, engineering, and mathematics and/or trades will be empowered and equipped to follow their pathways of choice.

8. **WE will develop** confident educational leaders in math as they collaborate on resource development and support and monitor best practice and actualization of curricula.

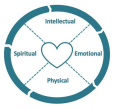
9. **SASKATCHEWAN will be** recognized as a leader in effective mathematics education.

[2]Saadi, K. (2002). *Abstract Thinking*. Life.ca. Retrieved 11 January 2021, from https://www.life.ca/lifelearning/1610/abstract-thinking.htm.

How to Read This Document:

This document contains the foundational pieces you will find on the SaskMath Web Resource. The content is rich and robust and we invite you to explore all that it has to offer. To create efficiency, this document is intended to be interactive. As such, while it is in PDF format, titles and sections are linked to connecting documents that provide deeper insights and resources. It is not recommended that this document be printed in its entirety but individual pieces may be printed as needed if you choose a print option. Be sure to click on underlined titles to see what else is contained in each section.

The following icons will appear throughout the resource to be your guide for ready reference:



Holistic/Indigenous Ways of Knowing Connections

Video

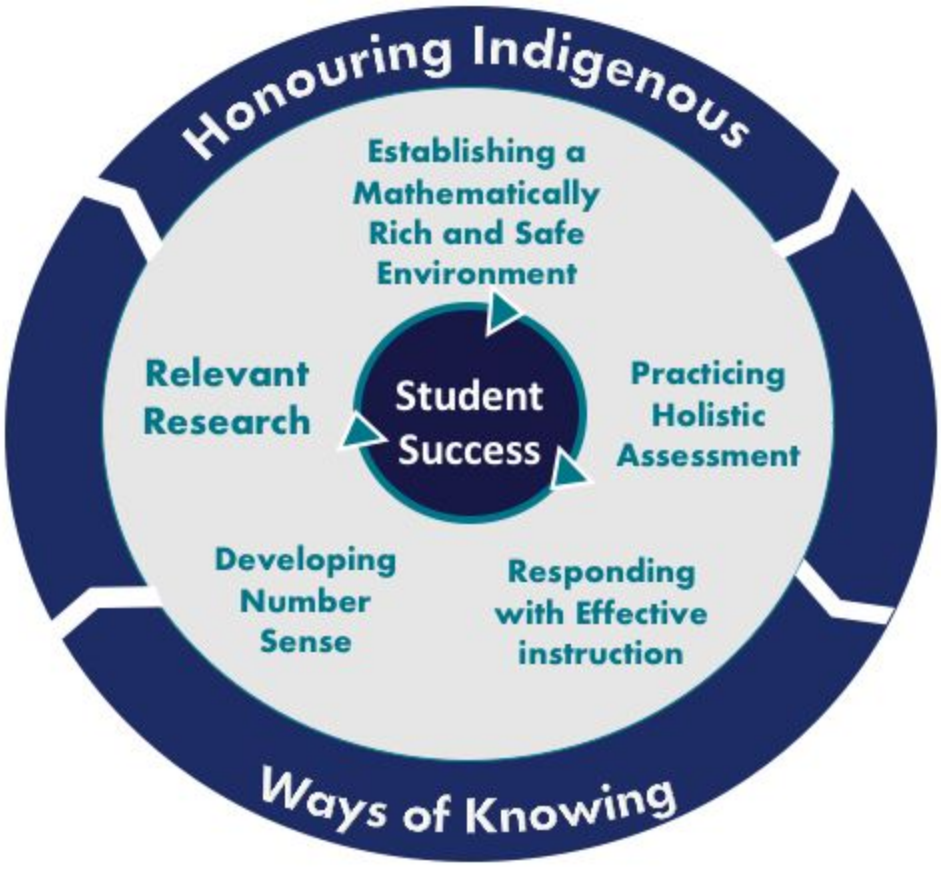
Questions for Reflection

Online Link

Mathematical Processes

Essentials of Math Instruction:

The following graphic captures the essence of what matters about teaching and learning mathematics:



Though the concepts here are listed, there is not a hierarchy in terms of priority. The components are interwoven and the approach is cyclical.

- Establishing a Mathematically Rich and Safe Environment
    - Interacting and collaborating during learning
    - Emphasizing mathematics as a human endeavour
    - Fostering relationships

- Providing an academically safe environment to take risks and make mistakes
- Sharing power, voice, and choice

- **Honouring Indigenous Ways of Knowing - Implementing the Student Success Document**

  - Honouring the four directions/teachings of the Medicine Wheel;
  - Creating care and connectedness - relationship within one's self, with others, and with the natural world;
  - Developing multiple pathways to learn and show what you know;
  - Emphasizing the importance of story and place; and
  - Sharing power, voice, and choice.

The Math teaching strategies in this Mathematics resource provide teachers with many options to weave ideas using indigenous ways of knowing/learning into their math lessons.
Myrna Turner, Cultural Coordinator, Muskoday First Nation Community School

- **Practising Holistic Assessment**

  - Triangulating data: conversations, observations, products
  - Honouring the learner (emotionally, spiritually, academically and physically)
  - Using assessment for, as, and of learning via a variety of strategies

- **Responding with Effective Instructional Practices**

  - Creating experiences for students to engage with number relationships
  - Sharing power, voice, and choice
  - Making sense through dialogue and interactive experiences
  - Teaching responsively (scaffold)

- **Developing Number Sense**

"The concept of life in numbers describes a relationship between the action, personality, and animation of numbers with the people using them in a given situation."[3]

(Barta et al., 2014, p.3)

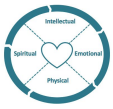
  - Relationships exist between and within numbers
  - Multiple instructional approaches are used to build student understanding of number relationships
  - Comparing, composing and decomposing, estimating, visualizing, and representing are used in connection with each other and are vital for developing number sense

[3] Barta, J., Eglash, R., & Barkley, C. (2014). Introduction. In J. Barta, R. Eglash & C. Barkley (Eds.), *Math is a verb: Activities and lessons from cultures around the world* (pp. 1-7). Reston, VA: National Council of a Teachers of Mathematics.

- **Integrating Relevant Research**

    - Works Cited
    - Bibliography
    - Resources to Share

"Over the last 15 years, researchers have shown that learning is not just about accumulating knowledge; it is about the formation of an identity … not a stable construct … but as a set of ideas, beliefs, and behaviors that may be performed in specific domains, such as the learning of mathematics" (Boaler & Selling, 2017, p. 82)[4].



Indigenous Ways of Knowing Connection to Conceptualizing the Essentials of Math Instruction

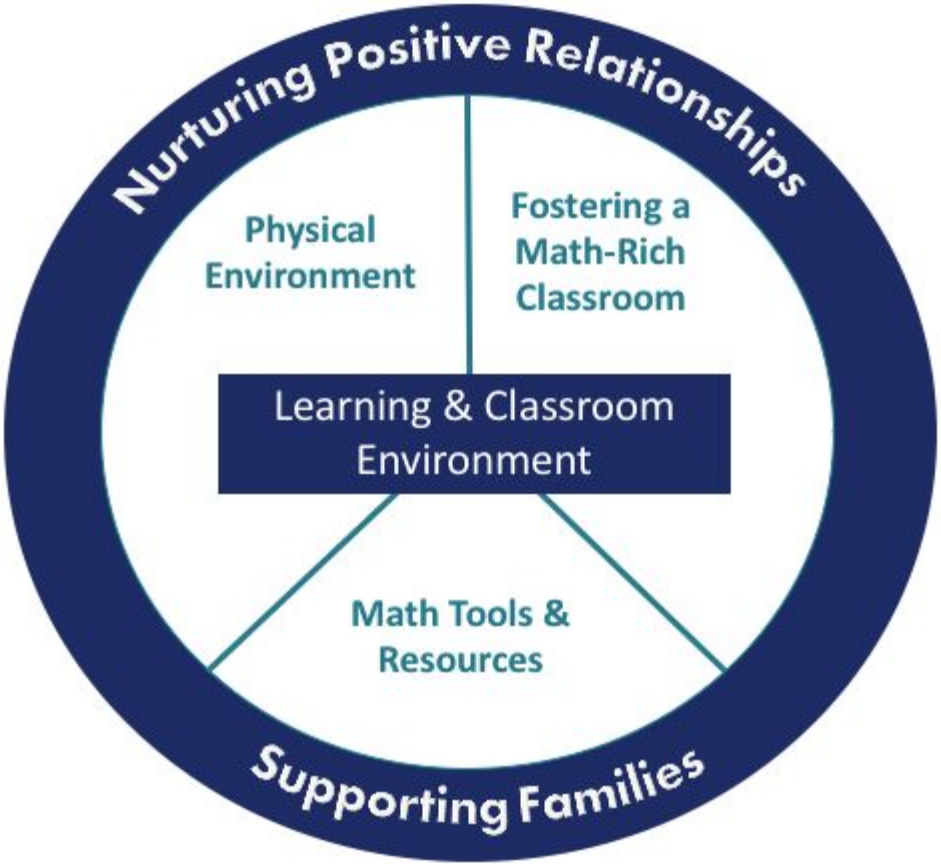
"Mathematics and culture come to be seen as integral components of one another."
(Barta et al., 2014, p. 4)

Questions for Reflection	• How are these concepts interwoven? • How will you simultaneously develop these big ideas into your practice?

[4]Boaler, J., & Selling, S. (2017). Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education, 48*(1), 78-105.

Establishing a Mathematically Rich and Safe Environment

Learning and Classroom Environment can be viewed through the following graphic representation. Positive Nurturing Relationships is intrinsic within all of the other three contributing facets: Physical Environment, Fostering a Math-Rich Classroom, and Math Tools and Resources.



I. Nurturing Positive Relationships:

There are many reasons to foster positive relationships in a Math classroom. It is important to create positive, nurturing relationships between student and teacher in addition to building relationships between and among peers.

I. Physical Environment:

"The environment…conveys the message that this is a place where adults have thought about the quality and instructive power of space. The layout of the physical space is welcoming and fosters encounters, communication, and relationships. The arrangement of structures, objects, and activities encourages choices, problem solving, and discoveries in the process of learning." [5]

II. Fostering a Mathematically Rich Classroom:

Approaching math with a holistic view of the student and the intent to encourage students to learn from mistakes, to persevere in overcoming challenges and to recognize and enjoy success, will invite students of mathematics to reach their potential. We need to communicate high expectations to all learners.

It is essential for the teacher to nurture a positive disposition toward the learning of math (i.e. a growth mindset) where students see themselves as capable learners.

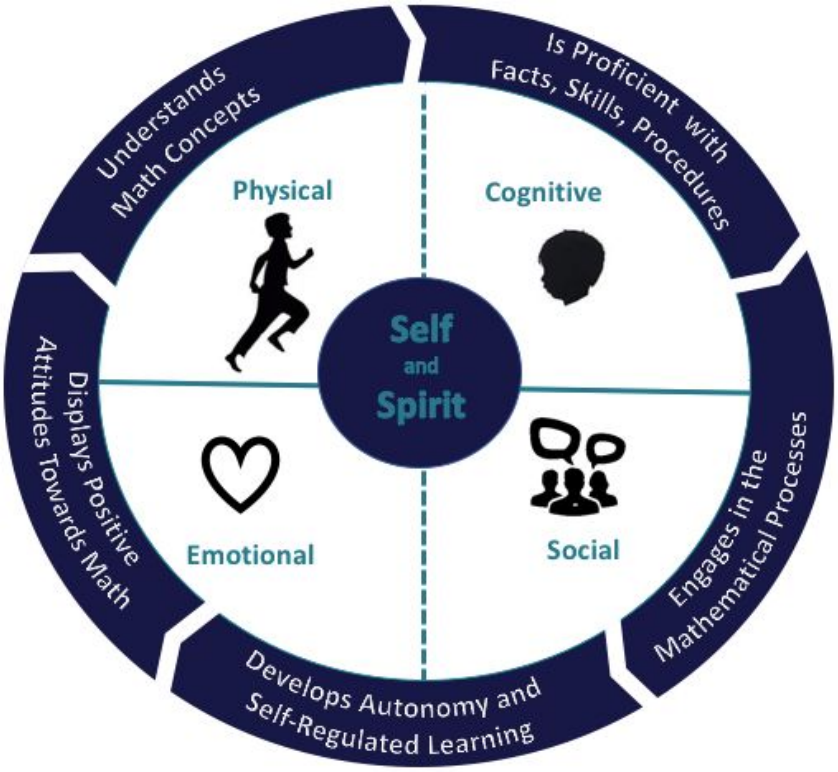
IV. Math Tools & Resources

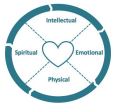
There are many kinds of manipulatives that are commercially available in addition to manipulatives which can be made from common objects that are purchased inexpensively or found in one's environment. Experiment and find manipulatives that work well for your students and are best suited for the mathematical concept.

Click here for chart showing the Uses and Applications of Math Manipulatives

Ultimately, the goal of establishing a mathematically rich environment is to foster a well-rounded mathematical learner as illustrated by the following infographic:

[5]Curtis, & Carter. (2020). Retrieved 5 July 2020, from https://pubsaskdev.blob.core.windows.net/pubsask-prod/86149/86149-86149-Creating_Early_Learning_Environments_(003).pdf (p. 13)





Indigenous Ways of Knowing Connection to Establishing a Mathematically Rich and Safe Environment

Supporting Families:

In Saskatchewan, we are committed to improving our students' achievement in mathematics. Our vision is that Saskatchewan children will be confident math learners who have a deep level of understanding of mathematics. This will serve them throughout their lives and provide them with many job and career opportunities. When students have this mathematical knowledge and skill set they will confidently and competently:

"Parent engagement matters. Study after study has shown us that student achievement improves when parents play an active role in their children's education, and that good schools become even better schools when parents are involved..."

Ontario Ministry of Education (2018)

- Solve problems and make sound decisions
- Calculate with ease
- Understand patterns and trends so that predictions can be made
- Manage time and money
- Handle everyday situations that involve numbers

Honouring Indigenous Ways of Knowing

This section follows the recommendations and guidelines of the *Inspiring Success: First Nations and Métis PreK-12 Education Policy Framework* document. Each section includes a detailed and comprehensive discussion with important links, resources and considerations for teachers. This important resource supports the recommendations of the Truth and Reconciliation Commission's Calls to Action. Teachers should feel supported through:

- Honouring Indigenous Ways of Knowing (past and present)
- Supporting Indigenous students in acquiring mathematical skills through understanding intellectual, cultural, socio-economic, political, and historical implications through the lens of Truth and Reconciliation.
- Enhancing math through the lens of historical ways of knowing and current pedagogy.

Click Here for Indigenous Ways of Knowing and Mathematics (Rationale)

Principles:

1. First Nations and Métis worldviews are valid ways of knowing and understanding the world;
2. Accountability in the area of First Nations and Métis education is essential to progress;
3. Communication throughout the system is key to the achievement of common goals;
4. Quality and authenticity are essential considerations in all First Nations and Métis education policy and program initiatives; and
5. All people must have equitable opportunities to succeed, coupled with respect for individual experiences and knowledge.

These follow the The Inspiring Success: First Nations and Métis PreK-12 Education Policy Framework (Saskatchewan Ministry of Education, 2018) which asserts that:

- First Nations and Métis Education… is a holistic, lifelong, culturally responsive and affirming learning that allows students to reflect on their relationships with themselves, one another and the natural world.

- First Nations and Métis Education… is a commitment to improving achievement for all learners by providing equitable opportunities for all to succeed and contribute to society in a meaningful manner.
- First Nations and Métis Education… comes from the teachings of Elders and Traditional Knowledge Keepers who reveal First Nations and Métis worldviews as valid ways of knowing and understanding the world.
- First Nations and Métis Education… incorporates First Nations and Métis ways of knowing as historical and contemporary cultures that are rooted in First Nations and Métis languages, and require the protection, revitalization and retention of languages in order to flourish and thrive.[6]

It is through the authentic implementation of these core principles that we will be able to affirm our pedagogy grounded in evidence-based practices.

The Saskatchewan Ministry of Education makes the following recommendations (for full explanation, click on the titles below):[7]

Recommendation 1: Cultural Affirmation and School Climate

Recommendation 2: Shared Decision Making

Recommendation 3: Curriculum Actualization:

- **Click here for Culturally Valid Assessments for Indigenous Learners**

Recommendation 4: Lifelong Learning

Other Resources:

- **Elder Protocol**[8]
  This protocol contains guidelines and suggestions when engaging with Elders.

[6]Saskatchewan Ministry of Education. (2018). First Nations and Métis Education | Saskatchewan Education and Learning | Government of Saskatchewan. Retrieved 21 July 2020, from https://www.saskatchewan.ca/residents/education-and-learning/first-nations-and-metis-education#inspiring-success; p.i.

[7] Saskatchewan Ministry of Education, F. (2009). A Time for Significant Leadership: Catalyst Leaders Toolkit. Retrieved 5 July 2020, from https://publications.saskatchewan.ca/#/products/74080

[8] From Indigenous Protocols, Indigenous Corporate Training Inc. (2019) www.ictinc.ca

- **Tobacco Offering**[9]
  The phrase "protocols with tobacco" refers only to the pre-contact meaning of the word tobacco, in spite of the fact that only post-contact manufactured tobacco is available to use. This document provides the background, knowledge and process that you need to be aware of in the ceremonial offering of Tobacco both formally (in offering it to Elders or officials) and informally (in offering it in ceremony to Creator).

- **Theresa's Story**[10]*:*
  Illustrates the benefits from Interacting with Elders and Knowledge Keepers. Based on her initial involvement with a year-long professional development program, Theresa shares some experiences and insights she gained at her high school that has a sizable Indigenous student population. Throughout the handbook, stories are shared by other teachers who participated as well. Their stories help bring the handbook's topics to life.

Practising Holistic Assessment: High Quality Assessment Practices

> *Assessāre* is a Late Latin frequentative verb derived from *assess,* the inflectional stem of the past participle *assessus,* from the Latin verb *assidēre* "to sit next to or by ". [11]

Assessment Of, As, and For Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessments and evaluation of student achievement must be based on the outcomes in the provincial curriculum and allow for flexibility determined

When we place an excessive value on test scores or grades, we communicate fixed mindset messages to our students, families and teachers see children as the words that label them; "high,"

[9] Aikenhead, G., Brokofsky, J., Bodnar, T., Clark, C., Foley, C., ...Strange, G. (2014). Enhancing school science with Indigenous knowledge: What we know from teachers and research. Saskatoon, Canada: Saskatoon Public School Division.

[10] Aikenhead, G., Brokofsky, J., Bodnar, T., Clark, C., Foley, C., ...Strange, G. (2014). Enhancing school science with Indigenous knowledge: What we know from teachers and research. Saskatoon, Canada: Saskatoon Public School Division.

[11] Definition of assess | Dictionary.com. (2020). Retrieved 6 July 2020, from https://www.dictionary.com/browse/assess

by the needs of the student. *"Mathematics is as much an aspect of culture as it is a collection of algorithms,"*[12] Assessment practices must be culturally valid as well as algorithmically valid.

There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

"average" or "low." Students quickly take ownership over their status. For example, students may take on the label of a "C" student and resign themselves to an incorrect belief that this reflects some innate mathematical capability and that working harder will not change that.

*(*Boaler, 2020).

Assessment for learning (formative assessment) involves the use of information about student progress to support and improve student learning, inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use;
- occurs throughout the teaching and learning process, using a variety of tools; and
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to families in support of learning.

Assessment as learning (formative assessment) actively involves student reflection on learning, monitoring of his/her own progress, and:

- supports students in critically analyzing learning related to curricular outcomes;
- is student-driven with teacher guidance; and
- occurs throughout the learning process.

Assessment for and as learning may address students' mathematical mindsets as this is a factor that impacts learning.

Assessment of learning (summative assessment) involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes;
- occurs at the end of a learning cycle using a variety of tools;
- provides the foundation for discussions on placement or promotion. Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, families/caregivers, and others about student progress; and to make informed decisions about the teaching and learning process. (Saskatchewan Ministry of Education, 2010b, p. 33)

[12] Carl Boyer (1906-1976) American historian of mathematics. Quoted from an unknown 1949 calculus textbook.

Screeners:

"Screeners" are formative assessments that have been developed to identify potential gaps in prerequisite skills. Many school divisions have developed useful screeners. SaskMATH is offering the screeners developed by Good Spirit School Division[13] as an example. Please ensure that you read the "Background" section before downloading or using any screener. These Screeners may be used as is or adapted to meet your needs.

Assessment: Using the Triangulation of Data

How much assessment evidence is enough? Since Saskatchewan math curricula contain learning outcomes by grade, the simple answer is: When the student has demonstrated clear understanding of the outcome. When considering assessment, it is important to focus on outcome attainment over product. What evidence do you have that a student has met the outcome? There is no timeline for when a student must demonstrate an understanding. Students have an entire school year to demonstrate achievement of outcomes.

In an effort to earn a decent grade in an abstract mathematics course, Simeonov (2016, pp. 442-443) pointed out that most students memorize without meaningful understanding. Students learn to hate mathematics, and then as families, they infuse their attitude into their children for elementary teachers to confront. [14]
(Duchscherer et al., 2019, p. 49)

Assessment Tools for Observation and Data Collection

Effective Feedback

Most student evaluations are composed of both evaluative feedback[15], which "judges student work," and descriptive feedback, "which provides information about how a student can become more competent." Studies

[13]The GSSD Math Screeners are a collaborative effort, created by Cindy Smith, Arlene Prestie, Michelle Pfeifer, Candice Gale, Tenille Flick and Lana Steiner and vetted by the GSSD Math Intervention Team.

[14] Culture-Based School Mathematics for Reconciliation and Professional Development | McDowell Foundation. (2020). Retrieved 5 July 2020, from http://mcdowellfoundation.ca/research/culture-based-school-mathematics-for-reconciliation-and-professional-development/

[15] Schinske, J., & Tanner, K. (2014). Teaching More by Grading Less (or Differently). *CBE Life Sci Educ.*, *13*(2). p. 159-166. https://doi.org/10.1187/cbe.CBE-14-03-0054

have found that while students focus heavily on evaluative feedback, descriptive feedback is much more important for student learning[16].

Consider one teacher's experience:

" For the last three years in each of my mathematics content courses for preservice elementary school teachers, I have instituted biweekly quizzes that did *not* carry a grade. These "quizzes" were short written assessments that asked students questions similar to those that would be on the course's summative assessments (midterms and final exams). They provided students with an opportunity to practice formalizing their thoughts on paper. I evaluated these quizzes for correctness by writing comments on the students' written responses but never assigned a numerical or letter grade. The comments asked students to explain their thinking further and to clarify their mathematical language; they also affirmed students' thinking when they provided thorough explanations, so that each student knew whether or not his or her understanding of the topic was sufficient.

The National Council of Teachers of Mathematics reminds us that assessment is an essential piece of classroom instruction and not just "the final stage in the traditional teach-learn-assess cycle" (NCTM 2014, p. 94).

...almost all students across all eight courses stated that they would not have preferred a grade on the assessment; they were able to focus on the feedback they received to deepen their understanding instead of being distracted by the (potentially poor) grade."[17]

John Hattie places feedback (Here stated as Evaluation and Self Reflection) at an effect size of 0.75. Click here for Hattie's 250+ Influences on Student Achievement. [18]

Summative evaluation is for reporting only. When used in isolation solely as an end point, summative assessments do not enhance student learning, and in fact can be damaging to student agency and self efficacy. Formative evaluation and assessment in all its forms promotes learning.

Click here for information with respect to Grading Vs. Assessing.[19]

[16]Pircon, B., Pircon, B., & Pircon, B. (2019). *The Importance of Feedback for Student Learning*. The Graide Network. Retrieved 22 January 2021, from https://www.thegraidenetwork.com/blog-all/importance-of-feedback-student-learning.

[17] Swartz, B. (2016). *Focus on Learning, Not Grades - National Council of Teachers of Mathematics*. Nctm.org. Retrieved 22 January 2021, from https://www.nctm.org/Publications/Mathematics-Teacher/Blog/Focus-on-Learning,-Not-Grades/.

[18] *Hattie effect size list - 256 Influences Related To Achievement*. VISIBLE LEARNING. (2018). Retrieved 22 January 2021, from https://visible-learning.org/wp-content/uploads/2018/03/VLPLUS-252-Influences-Hattie-ranking-DEC-2017.pdf.

[19] Information provided by Cindy Smith, Christ The Teacher School Division

A note about "spaced practice": Teachers may assign more tasks that are not necessarily evaluated but are intended to consolidate learning. This ensures that concepts are not taught in isolation, memorized and then forgotten. Evidence-based practice would assume that the student is able to demonstrate their understanding at any point in time. Spaced practice enables consolidation of learning. Key concepts are revisited throughout the year. Spaced practice may also provide multiple opportunities for students to demonstrate understanding over time.

Quality, balanced assessment is the goal, keeping in mind the importance of holistic assessment (gathering data from multiple perspectives), triangulation of data, and the importance of cultural relevance.

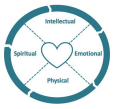
"...having a myriad of marks in the marks book is not necessarily a desirable objective."[20]

Alberta Assessment Consortium

Education Sector Strategic Plan (ESSP) Math Data Collection

When gathering data to inform the placement of students on the holistic math rubric, the intent is to look at multiple samples of work done throughout the year specifically within the number strand. Any of the assessment tools and strategies suggested in this section can, and should, be used to inform one's professional judgment to make a holistic determination of students' understanding.

- ➔ ESSP Provincial Math Rubrics[21]
- ➔ ESSP Math at Grade Level Documents[22]



Indigenous Ways of Knowing Connection[23]

Culturally Valid Assessment

Teacher interviews with students can draw on the recurrent learning strength of the oral tradition for communicating. Interviewing is an excellent assessment technique for culturally responsive mathematics teaching, as long as students believe that their teacher wants to find out what they do know.

[20]Alberta Assessment Consortium. Retrieved 9 July 2020, from https://aac.ab.ca/
[21]ESSP Provincial Math Rubrics. (2020). Retrieved 8 December 2020, from https://www.edonline.sk.ca/webapps/blackboard/content/listContent.jsp?course_id=_2869_1&content_id=_355613_1
[22]Education Sector Strategic Plan. (2020). ESSP Math at Grade Level Documents. Retrieved 8 December 2020, from https://www.edonline.sk.ca/webapps/blackboard/content/listContent.jsp?course_id=_2869_1&content_id=_356170_1&mode=reset
[23]While the heading indicates an Aboriginal context, the considerations in this section are universally applicable to a Holistic way of assessing all children.

One of the most promising practical strategies in a teacher's repertoire of culturally valid assessment is the portfolio (Click here for more details on portfolios).[24] Using portfolios nurtures students' responsible *autonomy* in the classroom, strengthens students' collaborative *relationships* with their teacher, and encourages students to develop a capacity for *self-assessment*. Click here for more on ePortfolios.[25]

When students are involved in the classroom assessment process, they become more engaged in learning. Anne Davies[26]

Portfolios can be used to draw upon students' cultural assets, and emphasize important cultural resources of Indigenous students, while greatly benefiting non-Indigenous students at the same time (i.e. artifacts as part of the collection of evidence).

Culturally invalid assessment occurs when culture clashes of learners are ignored; rather than being recognized and attempts made to ameliorate the clashes.

Responding with Effective Instructional Approaches

Curriculum Connections:

> "The aim of Saskatchewan's K-12 mathematics program is to help students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning." (Saskatchewan Ministry of Education, 2009)[27]

[24]Much of the discussion on portfolios, draws heavily upon the Saskatoon Public School Division's science professional development monograph: Aikenhead, G., Brokofsky, J., Bodnar, T., Clark, C., Foley, C., Strange, G. (2014). *Enhancing school science with Indigenous knowledge: What we know from teachers and research*. Saskatoon, Canada: Saskatoon Public School Division

[25] *ePortfolios Explained: Theory and Practice. Centre for Teaching Excellence, University of Waterloo.https://uwaterloo.ca/centre-for-teaching-excellence/teaching-resources/teaching-tips/educational-technologies/all/eportfolios*

[26]Davies, A., Herbst, S., & Reynolds, B.P. (2008). Leading the way to making classroom assessment work. Courtenay, BC, Canada: Connections Publishing.

[27]Saskatchewan Ministry of Education, 2009c, p.6

There are commonalities amongst all mathematics curricula in Saskatchewan. These important components drive high quality instruction across grade levels, communities of learners, and classrooms in our province.

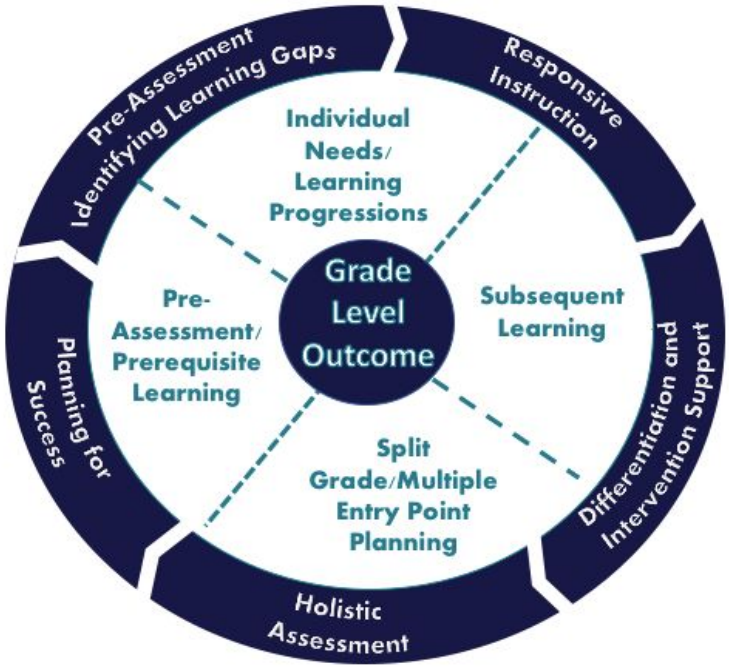
Click here for Curriculum Connections Document

Learning Stages and Scope and Sequence:

Students are continuously building upon their prior math knowledge to construct new understandings. As teachers, a solid understanding of the scope and sequence of mathematics learning is an essential component for responsive planning and instruction.

Many school divisions across the province have created their own versions of scope and sequence documents. We encourage you to seek out the scope and sequence documents in your division to use alongside or in place of these documents when planning for and supporting your students.

Saskatchewan has a diverse body of school and classroom demographics and as such, these documents are meant to support a variety of different applications such as the following:



1. **Supporting vertical planning** by providing a quick reference of the prerequisite knowledge for topics at each grade level and how that topic is built upon in future grades. By providing an understanding of how the mathematical concepts can be introduced and built upon,

teachers are able to structure the learning in their classroom to support students in building their mathematical understandings.

2. **Supporting teachers** with combined grade classrooms in coordinating topics of study. A reality in our province is that teachers are often teaching combined grade classes of two or more curricula. It is our hope that this document will provide teachers with support to build their year and lesson plans with coordinated topics and activities.
3. **Supporting interventions** for students that have gaps in their prerequisite knowledge. We know that when a student is missing one of these important pieces of the puzzle, it is essential to bridge the gap that exists in student prior knowledge before we can continue with new concepts. Occasionally, when a gap is identified in a student's prior knowledge, we struggle to find where this concept or skill was initially covered in the curriculum or where to find additional support to meet our students where they are in understanding.
4. **Determining appropriate course pathways** applicable for students. Having an understanding of how students progress through the curriculum, along with an understanding of post-secondary program requirements, will enable thoughtful decision-making when completing course registration and graduation plans.

Scope and Sequence Documents[28] (PDF's linked below)

- Ministry Scope and Sequence K-9
- Scope and Sequence K-9
- Scope and Sequence Gr. 8 - 12 (All Pathways)
- Scope and Sequence Gr. 8 - WA 30
- Scope and Sequence Gr. 8 - Foundations 30
- Scope and Sequence Gr. 8 - Calculus 30
- Scope and Sequence Gr. 8 - Math 21 (Modified Pathway)

Effective Instruction

AN EFFECTIVE MATHEMATICS PROGRAM

Effective mathematics instruction includes conceptual understanding, procedural fluency, problem solving, and its human dimensions. Students need to learn math in a logical, sequential manner[29], and commensurate with their maturity and personal experiences. Continual connections should always be made between past mathematical understanding

[28] Adapted from Greater Saskatoon Catholic Schools Concept Map with Basic Concepts of Math Correlation January 2015

[29] This is referring to the notion that additive thinking comes before multiplicative thinking, which precludes exponential thinking. This progression of thinking is reflected in the SK k-9 Math Curriculum.

and new concepts; connections between concrete experiences, pictorial representations, symbols, and algorithms. Teacher clarity and content knowledge are essential. Immersing students in relevant mathematics language and vocabulary, dialogue, experiential learning, collaborative construction of meaning, independent practice and reflection are all components of a balanced math approach.

Click here to learn how the 7 Mathematical Processes can impact your classroom.

Vocabulary and the language of mathematics:

Mathematics is a language of its own, with not only specific vocabulary but also symbols and a continual translation between mathematical notation and English language. Learning mathematics can be compared to learning in a bilingual setting, therefore the importance of building vocabulary must not be overlooked. Hattie notes that vocabulary programs in math have a significant impact (effect size 0.67)[30]. Indeed, we cannot expect students to deepen their understanding through mathematical discourse or to explain their reasoning if we do not give them the language to do so.

Marian Small reminds us that the thinking matters more than the vocabulary (M. Small (personal communication, January 25, 2021). See this link for more information with respect to developing classroom discourse.

> If math language isn't explicitly taught, children learn to disregard math words and only pay attention to the numbers.[31]

An effective vocabulary program involves student contributions as well as teacher modeling. The comprehension strategies that teachers use for literacy instruction also apply to literacy in math (making connections, summarizing, note taking, reading and writing responses, etc.). For example, mathematical word walls are effective if they are incorporated into the topic being taught and referred to continually. This is significant because commercial word walls can be purchased and posted without reference and context.

The Saskatchewan Math Curriculum[32] suggests that rather than teaching new math words explicitly, meaningful construction and understanding of mathematical terminology by students involves the students, individually

[30] Hattie, J. Visible learning for mathematics, grades K-12 (p. 153)

[31] https://luminouslearning.com/blogs/sped-math/teaching-math-vocabulary

[32] Grade 2 Curriculum Page 11

and then as a group, negotiating with the teacher the definitions to be used within the class. At that point, the students are then prepared to consider published definitions and to read and critique them.

Possible Approaches to Vocabulary Instruction[33] (link takes you to source for reference)

Generative vocabulary instruction[34] refers to highlighting root words and sources of words for students, and discussing where they may encounter those words outside of math class. It involves making and strengthening connections. For example: quadrilateral, quadratic, quad (ATV with 4 wheels), quadriceps (a muscle group of 4), quadrant, quadruple, etc. Expanding vocabulary exploration across other subject areas will strengthen students' understanding. Frontloading vocabulary is an effective way to provide pretext and connections.

Because math involves translating back and forth between spoken language to mathematical symbols (for example *decreased by, less than, diminished, difference*, etc), consider introducing symbolic writing with a "we write" and "we say" notation.

For example ***we say*** "four less than a number is seven" and ***we write*** $n - 4 = 7$

For example, ***we write*** $\sum_{n=1}^{6} 4n$ and ***we say*** "Find the sum of all the terms generated by the

expression 4n, starting at the first term, where n=1, and stopping at the 6th term".

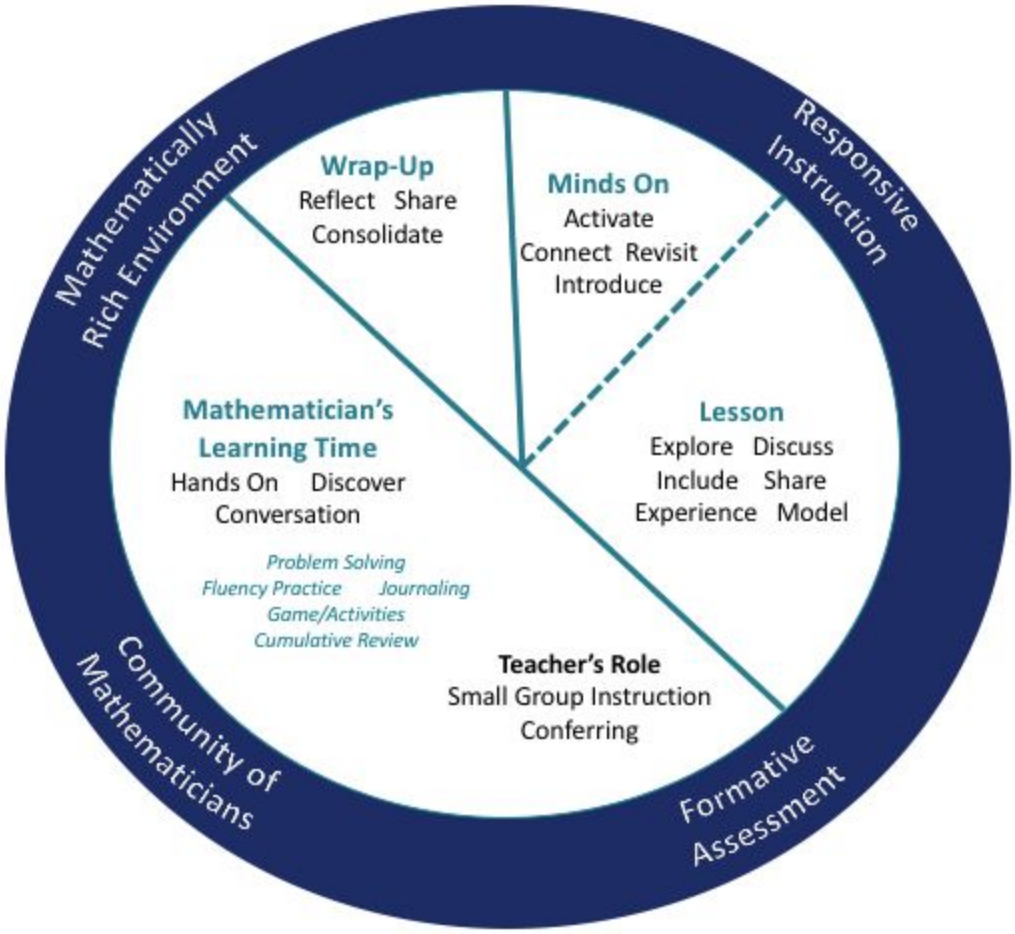
CLICK TO ENTER THE INTERACTIVE MATHEMATICS GLOSSARY.[35]

[33] Uscianowski, C. (2018). Use these 5 actionable strategies today to grow your students' math vocabulary. Retrieved 10 December 2020, from https://luminouslearning.com/blogs/sped-math/teaching-math-vocabulary

[34] Templeton, S., Bear, D., Invernizzi, M., & Johnston, F. *Vocabulary their way*. Pearson.

[35] Learn Alberta. Interactive Mathematics Glossary. Retrieved 5 July 2020, from http://www.learnalberta.ca/content/memg/1_A/index.html

Mathematician's Workshop or Framework (Click here for details)[36]



This instructional framework guides teachers to provide opportunities for whole class, small group, partner, and individual learning. Students are more engaged, grow in their mathematical understanding and see themselves as mathematicians when they collaborate in their learning.

Pedagogical Models in a Mathematics Context:

An effective mathematics program balances many pedagogical tools in response to student needs and relies on the professional judgment of the

[36] Adapted from Saskatoon Public Mathematician's Workshop Framework, 2017

teacher. In a mathematics classroom, there is a balance between explicit instruction and inquiry. While universal principles of high quality instruction apply to mathematical contexts, there are some principles that are unique to mathematics that are necessary to understand.

Differentiation through Responsive Pedagogy

In the Saskatchewan context, differentiation is addressed through the Adaptive Dimension[37] which enables all teachers to respond to student diversity, including their strengths and needs, interests, backgrounds, life experiences and motivations. The Adaptive Dimension refers to the concept of making adjustments to any or all of the following variables: learning environment, instruction, assessment and resources. Adjustments to these variables are intended to make learning meaningful and appropriate and to support student achievement. Tomlinson (1999) states, "Differentiation is an organized yet flexible way of proactively adjusting teaching and learning to meet kids where they are and help them to achieve maximum growth as learners" (p.14).[38]

"The first line of instruction is always the classroom. No series of interventions - even highly effective ones - can take the place of good classroom instruction that builds a rich base and creates a community of learners"

(Fountas & Pinnell, 2009, p. 497)

Fundamental to differentiation is the understanding that one does not alter the outcome but rather provides alternatives in instruction, assessment, and/or environment to meet the needs of the learner.

Differentiating Is...	Differentiating Is Not..
Providing choice as much as possible. Students can choose: ● Manipulatives (choosing which manipulative to use or choosing not to use any) ● How to show or represent their work ● The strategy for solving the problem	Requiring all students to use the same materials and follow the same procedures and steps.
	Providing tasks that students are successful at but fail to push their learning forward.

[37] Retrieved 2 December 2020, from http://publications.gov.sk.ca/documents/11/15309-Adaptive-Dimension.pdf

[38] Government of Saskatchewan, M. (2017). Adaptive Dimension. Retrieved 30 November 2020, from http://publications.gov.sk.ca/documents/11/15309-Adaptive-Dimension.pdf

Responsive Pedagogy: ● Using open-ended questions, rich tasks or low-floor, high-ceiling tasks so that all students have an entry point and the challenge level can be easily adapted. ● Providing choice and parallel tasks that demonstrate understanding in different ways through complex and creative tasks that challenge all students. ● Enriching learning by providing questions, tasks, projects and or research that allows for deeper learning of concepts beyond the curriculum. ● Leveraging technology to provide alternative tasks, target gaps in skills and knowledge, and provide more practice. Responsive Instruction: ● Allowing more time when students need it. ● Arranging small group instruction to target the needs of specific groups. ● Providing more explicit instruction and worked examples for struggling learners in an effort to help students learn to work more independently. ● Explicitly teaching and reinforcing self-coping strategies to create independence. Responsive Assessment: ● Prioritizing understanding and process over product.	Accepting incomplete work because students didn't have time to finish. Using only full class instruction. Talking students through every task so they don't have a chance to develop self-coping strategies. Allowing struggling learners to over-rely on extra supports and inefficient strategies. Giving the same task to all students. Only giving choice to students that would benefit from additional challenges. Using only closed tasks and questions (those with only one right answer). Allowing only high achieving students to participate in open or creative tasks. Rewarding quick and efficient completion of practice with more work. Teaching next year's outcomes. Using technology to entertain and occupy learners.

The Dufour's Professional Learning Community questions are helpful to guide a process for differentiation (Saskatchewan Reads Team, 2015)[39].

Ideas to Support Responsive Instruction

Ideas to Support Differentiation

? *Questions for Reflection*	• How have I differentiated the instructional approaches in my classroom to meet the individual needs of my students?

Intervention:

Intervention is a team approach that involves targeted data-driven actions to support learners who are not responding to the high quality, responsive instruction of the teacher. Differentiated

[39] Saskatchewan Reads Team. (2015). Saskatchewan Reads. Retrieved 5 August 2020, from https://saskatchewanreads.wordpress.com/ The Dufour model speaks to what students are expected to **learn**. The SK Math language is what students are expected **to know, understand** and be **able to do**.

instruction **and** intervention assume that not all students learn at the same pace and in the same way. Some students respond to a specific pedagogy more quickly and some students need a different approach to reach understanding. For any given goal or outcome, some students need more time and/or different types of support in order to learn and access grade level content. Others need support to fill in learning gaps that prevent them from accessing grade level content. Differentiated instruction is a *responsive pedagogy* that provides different materials, arrangements, and strategies equitably to meet the varying needs of students. Intervention is a *strategic and deliberate action* to support learners with an identified learning need that may be holding them back.

Intervention differs from responsive instruction in that it makes provision for additional time and support in a targeted and strategic manner. Typically, it is a short term support structure that may involve an individual or small group of students with a similar need. It is intended for students who, in spite of the high quality, responsive instruction in the classroom, are still not able to access grade level outcomes. Intervention is based on specific learning targets identified through quality assessment practices. The team will deeply analyze the data available and then set a strategic plan that will be implemented as a short term intervention. Intervention emphasizes the importance of "assessing learners' successes and needs *continually*."[40] As such, a specific time (5-6 weeks) should be designated and the success of the intervention should be frequently assessed.

An Intervention process IS[41]	An intervention process IS NOT
A process	A program
Sustained and ongoing support	A quick fix
Individualized to student need	More of the same intervention provided in classroom
Data-based (relying on progress monitoring and diagnostic data)	Based on anecdotal information
For a small subset of students	For all students who score poorly on a screening measure

[40] Seifert, K. Differentiated instruction and response to intervention | Educational Psychology. Retrieved 2 December 2020, from https://courses.lumenlearning.com/educationalpsychology/chapter/differentiated-instruction-and-response-to-intervention/

[41] National Center for Intense Intervention. Retrieved 3 December 2020, from https://intensiveintervention.org/intensive-intervention-math-coursehttps://intensiveintervention.org/subject/mathematics https://www.apbs.org/sites/default/files/conference-2016/presentations/j13-NCII-Website-Handout-apbs2017.pdf

The Institute for Educational Sciences in their document ***Assisting Students Struggling with Mathematics: Response to Intervention (RTI) for Elementary and Middle Schools***[42] identify eight research-based recommendations for math interventions:

Recommendation 1.

- Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

Recommendation 2.

- Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.

Recommendation 3.

- Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Recommendation 4.

- Interventions should include instruction on solving word problems that is based on common underlying structures.

Recommendation 5.

- Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Recommendation 6.

- Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Recommendation 7.

- Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Recommendation 8.

- Include motivational strategies in tier 2 and tier 3 interventions.

Click here for Tiered Intervention in Math

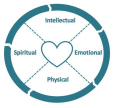
[42] Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J., & Witzel, B. (2009). Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Retrieved 3 December 2020, from https://ies.ed.gov/ncee/wwc/Docs/PracticeGuide/rti_math_pg_042109.pdf

Considerations for intervention team:

- Who is on your intervention team?
    - Who will need to be informed about the intervention?
- What data sources are you using to identify students who are struggling?
    - What data is informing the specificity of the targets and goals you set for students?
    - What do you need to get curious about with respect to this learner? What further assessment data is needed?
- What instructional strategies have already been employed?
    - What research based strategies can you try to support this learner?
    - Who is best to deliver intervention support?
        - Where else can you get support for this or additional strategic actions?
- How will you measure the success of the intervention?
    - How frequently will you assess the efficacy of the intervention provided?
    - What tools or methodology will you use?
- What is standing in the way of providing additional time and support?
    - What resources will you need?
    - What level of expertise is available?
    - What professional development is needed for the teacher or specialist supporting this learner?
    - How can your administrator (or division) help you structure time to provide these strategic actions? (i.e. creative time-tabling or staffing considerations.)

Considerations for Administrators and Division Personnel: A Word about Intervention Specialists

? *Questions for Reflection*	• What do I know about the process within my school for assisting struggling students? Who would I check with? • How have I engaged families/caregivers in the decision making around interventions for their child? • How am I monitoring the effectiveness of the intervention?



Click here for links to Indigenous Ways of Knowing Connection to Responsive Pedagogy

[Click Here for Indigenous Ways of Knowing Connection to Effective Instructional Practices]

Role of Technology:

How can technology support the mathematics classroom?

Saskatchewan Curriculum - Mathematical Processes:

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

> Strategic use of technology strengthens mathematics teaching and learning. Strategic use does not imply continuous use of technology. Rather, it refers to teacher and student use of technology tools as determined through decision making that keeps mathematics, and not technology, as the focus of instruction. (NCTM, 2015, p.1)

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- develop personal procedures for mathematical operations;

- create geometric displays;
- simulate situations;
- develop number sense;
- develop spatial sense; and
- develop and test conjectures.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Questions for Reflection

- Am I using this technology to enhance the learning of specific curricular outcomes?
- Will this technology make it possible to do more than I was able to do before?
- Can this technology make my work more efficient, and improve student achievement?

SaskMATH will not be able to maintain an up-to-date curated list of tools. The suggestions given are available at this time and search words are given to enable you to find similar tools in the future. There are a few cautions we would give you, with regards to the use of technology:

- Be sure to follow directions provided by your school division technology division with regards to acceptable apps, programs and web resources.
- Be careful to follow your school division guidelines regarding installation and use of apps to ensure the privacy of your students.

Technology must always be tied directly to learnings that support specific outcomes. (Eg: a daily routine of 15 minutes on Mathletics would not be supported.) Click titles below for more information.

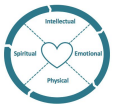
Assessment Technology

Digital Classroom / Communication

Digital Tools

Calculators in the Classroom

Teaching Strategies



Indigenous Ways of Knowing Connection: Technology/Games

Developing Number Sense

SASK MATH

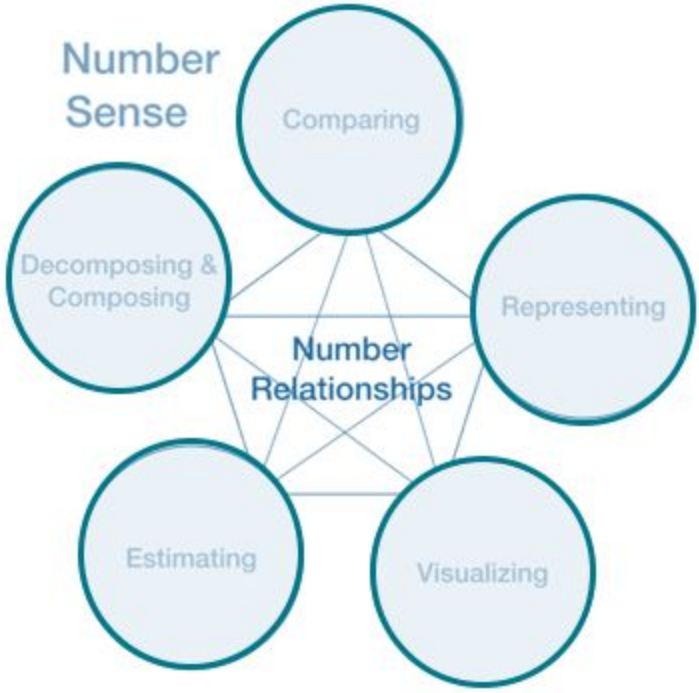
Developing Number Sense

- Relationships exist between and within numbers
- Multiple instructional approaches are used to build student understanding of number relationships
- Comparing, composing and decomposing, estimating, visualizing, and representing are used in connection with each other and are vital for developing number sense

Number sense is built through developing an understanding of number relationships. Providing all students with learning opportunities to compare, compose and decompose, represent, visualize, and estimate lays the foundation for a deep understanding of relationships between and within numbers. The provincial outcomes reflect these number relationship concepts. They are critical for student achievement of outcomes throughout all math curricula.

"Through their learning of K-12 Mathematics, students should develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems."

SK Curriculum Guide, Gr. 2, p. 7



Rarely are the essential components of number sense (comparing, estimating, visualizing, representing, decomposing and composing) explored in isolation of each other. Instead, students understand mathematics through experiences that involve many of these concepts. For example, students can estimate while using visuals. Students visualize an estimation. Students compose and decompose numbers using a visual representation. Students that understand number relationships also understand the relationships that exist within operations. For example, the relationship between addition and subtraction, multiplication and division.

Students' experiences with mathematics should not solely be watching the teacher and repeating procedures. This does nothing to build number sense, but instead just establishes classroom routines that are counter-productive to developing student number sense. Teachers facilitate experiences where students are engaged in productive struggle while thinking mathematically in order for students to become “numerate” (in the same sense that students become literate through struggling with experiences that strengthen reading and writing).

Integrating Instructional Approaches and Within Number Sense

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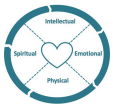

Indigenous Ways of Knowing Connection to Developing Number Sense

Integrating Relevant Research

Works Cited

Bibliography